Re-scaling the Battjes-Janssen model for depth-induced wave-breaking

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Abstract

The scaling of the energy dissipation of random, short-crested waves due to depth-induced breaking is investigated with a numerical model and a large number of laboratory observations and generalized lake observations. The scaling is found to depend on the normalized water depth $\bar{k}d$ (where d is water depth and $\bar{k}$ is an average wave number) and bottom slope n. The model has been supplemented with a relaxation model to represent the persistence of wave breaking, in particular at the steep edge of reefs. The $n-kd$ scaling does not improve the results for cases with gently sloping bathymetry (between 1:30 and 1:100, say; compared to using a commonly used fixed scaling). It does improve the results for flat bottom cases (lakes and reefs), reducing the rms-error in the significant wave height for these cases by over 50%.

1. Introduction

The Battjes-Janssen model is a widely used model to compute the reduction of wave energy as the waves travel through the surf zone (Battjes and Janssen, 1978). It is basically a bore model for each breaking wave, combined with the statistical characteristics of wave heights of random waves. It requires an estimate of the maximum possible individual wave height under the given conditions of breaking. In shallow water this maximum is usually expressed as a fraction γ of the local depth. On gently sloping beaches, a fixed value $\gamma \approx 0.73$ usually gives very reasonable results. However, this is not the case over horizontal bottoms when either considerably higher values are needed (lakes) or considerably lower values (reefs). Here we explain this dichotomy and we use observations in shallow lakes and 1D laboratory wave flumes to formulate a dependency of γ on normalized water depth (normalized with wave length) and bottom slope.

2 The wave-breaking model

Battjes and Janssen (1978) estimated the dissipation of a single breaking wave as for a bore and combined this with a clipped Rayleigh distribution to estimate the bulk dissipation of random waves

$$\varepsilon = -\tfrac{1}{4}\alpha \bar{f}\, Q_b \rho g\, H_{max}^2 \tag{1}$$

where $\alpha \approx 1$ is a tunable coefficient (we use $\alpha = 1$), $\bar{f}$ is the mean wave frequency, g is gravitational acceleration, ρ is the density of water, and H_{max} is the maximum possible wave height under given breaking conditions and Q_b is the fraction of breakers. This fraction depends on the root-mean-square wave height H_{rms} and H_{max}:

$$\frac{1-Q_b}{\ln Q_b} = -\left(\frac{H_{rms}}{H_{max}}\right)^2 . \tag{3}$$

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Others have suggested refinements of this model (e.g., Thornton and Guza, 1983; Van der Westhuysen, 2009, 2010) but these do not solve the dichotomy over horizontal bottoms. Beji and Battjes (1993) observed that the shape of the wave spectrum seemed to be unaffected by surf breaking and Booij et al. (1999) accordingly used a spectral distribution of the dissipation that is proportional to the spectral density of the spectrum:

$$S_{surf}(\sigma,\theta) = \varepsilon\, E(\sigma,\theta)/E \tag{5}$$

in which E is the total wave energy, $E(\sigma,\theta)$ is the energy density, σ is (spectral) frequency and θ is (spectral) wave direction.

In shallow water, when depth-induced breaking dominates dissipation, we take $H_{max} = \gamma d$. Many investigators have suggested relationships between γ and various characteristics of the waves and the bottom, for instance the initial wave steepness or the local bottom slope. None of these solve the above dichotomy. Based on comparing a large number of observations (see below) with a large number of trial computations with the various suggested formulations, we propose the following. Breaking occurs occasionally when waves approach shallow water. As the waves continue propagating, breaking increases as the value of γ decreases with decreasing relative depth ($\tilde{k}d$, in which $\tilde{k}$ is a characteristic wave number). In shallower water, the bottom slope n becomes relevant. Eventually, in very shallow water, the waves behave more and more as solitary waves and the normalized wave number $\tilde{k}d$ becomes irrelevant. The value of γ then depends only on bottom slope. For situations with a constant bottom slope, the value of γ (denoted as γ_{n-kd}) would therefore develop qualitatively as in Fig. 1.

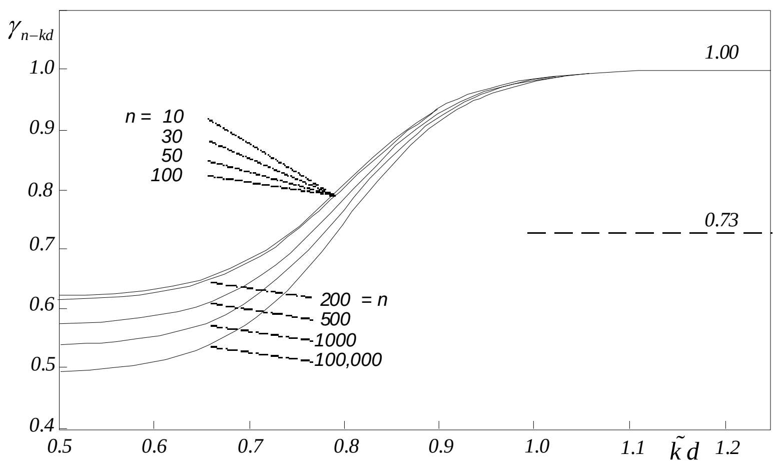

Fig. 1 The calibrated value of γ_{n-kd} as a function of bottom slope n and normalized depth $\tilde{k}d$, with $\gamma = 0.73$ as reference.

Over negative slopes the precise (relatively high) value of γ seems to be irrelevant and we use $\gamma = 1$. In cases with a varying bottom slope, our inspection of the observations suggests that the waves do not respond instantaneously to such variations, i.e., the value of γ is not locally determined – some degree of persistence seems to be involved. To accommodate this, we use a simple relaxation model for γ, with a persistence length depending on the local depth and bottom slope.

To calibrate and verify the model, we use the third-generation wave model SWAN (Booij et al., 1999) which is based on the balance equation of the wave action density spectrum. It accounts for propagation in geographical space, including depth-induced and current-induced refraction, shifting of the relative frequency due to variations in depth and currents, and wave generation by wind, nonlinear wave-wave interactions and dissipation. Our computations for idealized situations (laboratory and generalized lake observations) were carried out in 1D mode with only shoaling, bottom friction, triad wave-wave interactions and depth-induced breaking active (wave set-up is included in the computations).

3 Calibration and verification in idealized situations

We compiled a fair number of published observations taken in laboratory flumes (Fig. 2; Dingemans et al., 1986; Van der Meer et al., 2000; Jensen, 2002; Katsardai and Swan, 2011a,b; Smith, 2004, Battjes and Janssen, 1978 and Boers, 1996, 2005) and in the field: Lake George in Australia (Fig. 3; Young and Babanin, 2006), Lake Sloten and Lake IJssel in the Netherlands (Fig. 3; Bottema and Van Vledder, 2009).

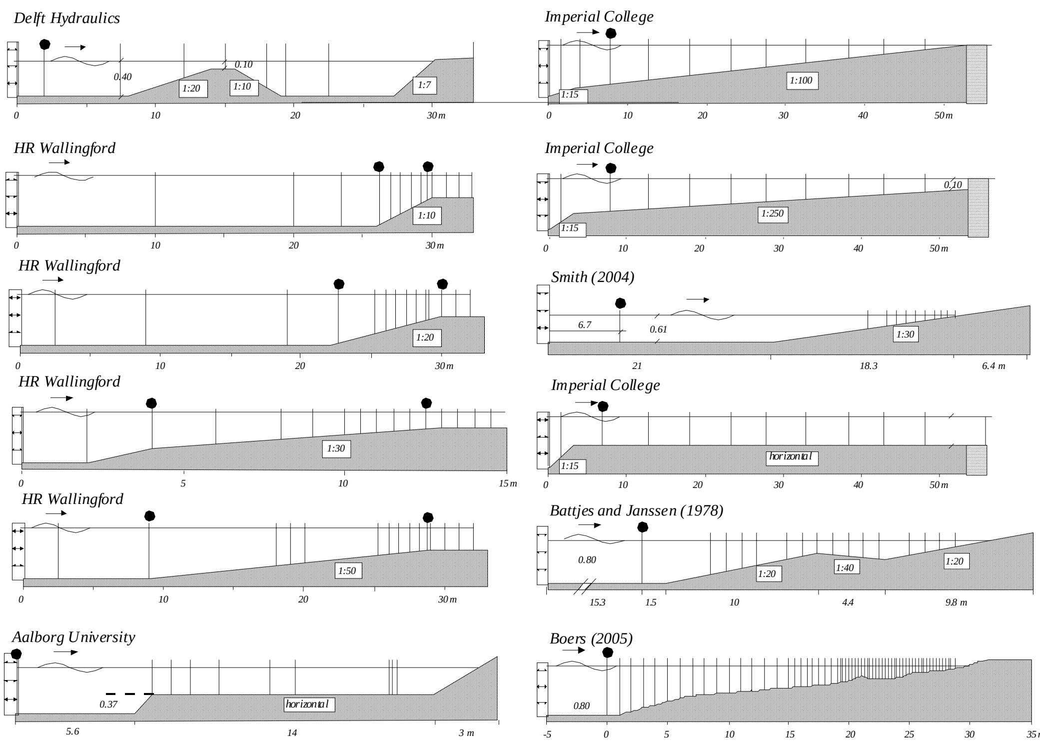

Fig. 2 The configuration of the laboratory observations (calibration and verification). Thin vertical lines indicate wave gauge positions. Solid dots indicate location of incident spectra (wave boundary condition) for the computations.

For the calibration with the laboratory observations, we selected from each of the laboratory sets, cases that represented the centre and the envelope of the experimental parameter values of that set (for instance, incident significant wave height, period, bottom slope, spectral peakedness, depth). Of the potential 121 envelope cases thus identified 80 were actually available and used for the calibration. For the calibration with the lake observations, we note that over the range of observed dimensionless depth $\tilde{d} = gd/U_{10}^2$, the expression of Young and Babanin (2006) approximates the upper limit of the data (dimensionless wave energy $\tilde{E} = g^2 E/U_{10}^4$, in which E is the total energy or the zero-th order moment of the wave spectrum) reasonably well (Fig. 4), except for the highest wind speeds when $\tilde{d} < 0.04$ and the limiting value seems to transit to the line $H_{m0}/d = 0.45$. A similar transit occurs in the SWAN computations with $\gamma = 0.73$ but to lower levels of H_{m0}/d . We selected the two observations nearest to $H_{m0}/d = 0.45$

for the calibration. The three next nearest data points are used for the verification. The results of the calibrations in the flumes and the lakes are shown in Figs. 1 and 4 (inset).

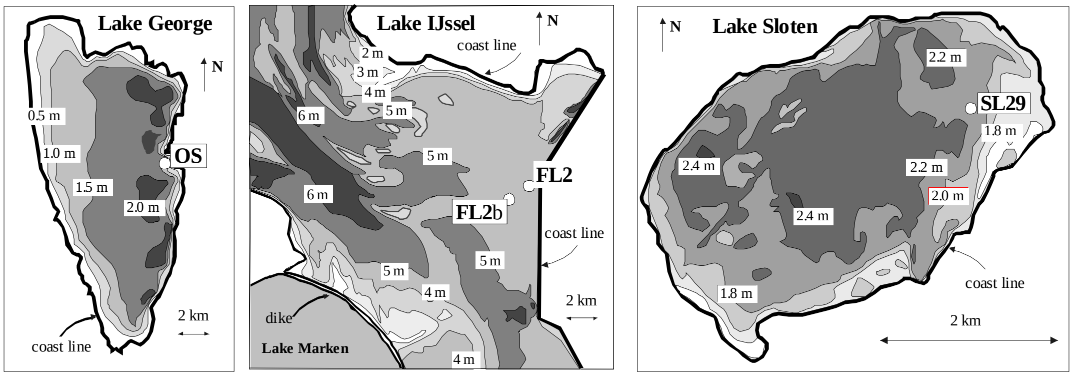

Fig. 3 The bathymetry of the three lakes used for the calibration. The location of observation sites are indicated with OS, FL2, FL2b and SL29.

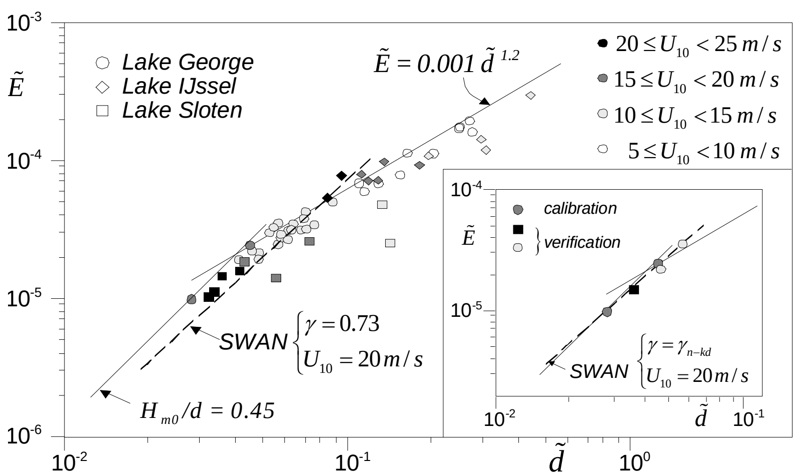

Fig. 4 The observed values of the dimensionless wave energy as a function of the dimensionless water depth for fully developed waves in shallow water of Young and Babanin (2006) and Bottema and Van Vledder (2009). Inset shows calibration and verification results.

To verify the $n-kd$ scaling with the laboratory cases, we removed from the compiled data set all cases that were used in the calibration and from the remainder randomly selected 125 cases. These verification results showed no improvement over gently sloping bottoms - the scatter index (the rms-error normalized with the average of the observed values) of the significant wave height remained at 8%. However, over the reefs (with relatively low kd values) the scatter index decreased from 14% to 8% and in the lakes (with relatively high kd values) from 16% to 2%.

4 Summary and Discussion

Using a fair number of observations in 1D laboratory flumes and in shallow lakes, and the random bore model of Battjes and Janssen (1978), we developed a scaling of energy dissipation of waves breaking in shallow water. When the waves arrive from the far field and travel over a horizontal bottom (corresponding to swell arriving at a reef), we found the proposed scaling to considerably improve the prediction of the significant wave height, at least under laboratory conditions. The rms-error reduced by 40% (compared to using a constant value $\gamma = 0.73$). When the waves are locally generated over a horizontal bottom (corresponding to wind waves in a shallow lake or over tidal flats), we found an 85% reduction of the rms-error. The $n-kd$ scaling does not reduce the rms-error for waves travelling over sloping bottoms.

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